# Experiment No. 4

**Title: Free damped oscillation of mass-spring system**

**Aim:** To obtain damped time period, angular frequency, frequency, logarithmic decrement, damping ratio, natural frequency, initial displacement, initial velocity, initial phase, amplitude, exponentially decaying envelop, u(t), v(t) and a(t)

# Diagram:

**Theory:** The displacement of a free damped spring mass system is given by

𝑢(𝑡) = 𝑒−𝜁𝜔𝑛𝑡(𝑢0sin (𝜔𝐷𝑡 − 𝜙0))

where, the amplitude 𝑢0 and phase 𝜑0 are given by

𝑢̇ (0) + 𝜁𝜔 𝑢(0) 2

𝑢0

( )

= √ 𝑛

𝜔𝐷

+ (𝑢(0))2

𝜙0

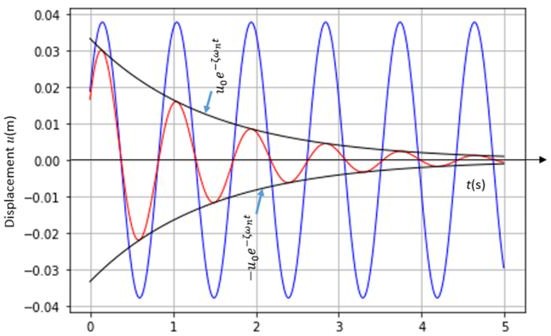
−1 ( −𝑢(0)

𝑛

= 𝑡𝑎𝑛 𝑢̇ (0) + 𝜁𝜔 𝑢(0) )

( 𝜔𝐷 )

The typical displacement versus time curve look like this:



The velocity and acceleration are 𝑣(𝑡) = 𝑢̇ (𝑡) and 𝑎(𝑡) = 𝑣̇(𝑡). The logarithmic decrement is

1 𝑢𝑖

𝛿 = (

𝑗

) 𝑙𝑛 ( )

𝑢𝑖+𝑗

And the damping ratio can be calculated from the logarithmic decrement from the relation:

2𝜋𝜁

𝛿 =

√1 − 𝜁2

The angular frequency *ωD* of a damped system is related with the natural angular frequency *ωn* as

𝜔𝐷 = 𝜔𝑛√1 − 𝜁2

# Procedure:

1. Set the spring mass system into oscillation and acquire the data digitally. Store the data of time and displacement in a file (Excel in our case).
2. Plot the displacement versus time curve.
3. Find the time period TD of the damped oscillations. Calculate 𝑓𝐷

= 1

𝑇𝐷

and 𝜔𝐷

= 2𝜋𝑓𝐷.

Calculate 𝜔𝑛

= 𝜔𝐷 .

√1−𝜁2

1. Identify u(0) from the data. Calculate v(0) by calculating slope from 𝑢̇ (0) = 𝑢(𝑡1)−𝑢(0).

𝑡1−0

1. Calculate 𝑢0 and 𝜙0.
2. Plot u(t) from the obtained parameters on the same graph as the raw data. Also plot the envelop

𝑢0𝑒−𝜁𝜔𝑛𝑡 and −𝑢0𝑒−𝜁𝜔𝑛𝑡. Comment on the deviations between the raw data and the theoretical curve which arises due to experimental error.

1. Find the velocity and acceleration numerically and plot it versus time.
2. Note the poor quality of velocity and acceleration graphs which highlights the importance of data quality.

# Observation:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Time(ms)** | **Distance(cm)** | | **Velocity v(t)** | | **Acceleration a(t)** | | **uoexp(-ζωnt)** | | **Negative Envelop** | | | **sin(ωD-φo)** | **Calculated uo** |
| 233 | 14.69 | | 0.804347826 | | 3.565904795 | | 0.023245031 | | -0.023245031 | | | -0.034342425 | -0.000798291 |
| 256 | 16.54 | | 0.886363636 | | -3.135106001 | | 0.023231195 | | -0.023231195 | | | 0.608528366 | 0.014136841 |
| 278 | 18.49 | | 0.817391304 | | 2.603540127 | | 0.023217968 | | -0.023217968 | | | 0.958703264 | 0.022259142 |
| 301 | 20.37 | | 0.877272727 | | -4.080578512 | | 0.023204148 | | -0.023204148 | | | 0.559558735 | 0.012984084 |
| 323 | 22.3 | | 0.7875 | | 1.846590909 | | 0.023190937 | | -0.023190937 | | | -0.573791201 | -0.013306756 |
| 347 | 24.19 | | 0.831818182 | | -5.23415978 | | 0.023176533 | | -0.023176533 | | | -0.705589387 | -0.016353116 |
| 369 | 26.02 | | 0.716666667 | | 0.441919192 | | 0.023163337 | | -0.023163337 | | | -0.914130244 | -0.021174307 |
| 393 | 27.74 | | 0.727272727 | | -6.353305785 | | 0.023148951 | | -0.023148951 | | | -0.831182041 | -0.019240992 |
| 415 | 29.34 | | 0.5875 | | -1.47192029 | | 0.023135771 | | -0.023135771 | | | 0.22188812 | 0.005133553 |
| 439 | 30.75 | | 0.552173913 | | -4.347826087 | | 0.023121401 | | -0.023121401 | | | 0.385618002 | 0.008916028 |
| 462 | 32.02 | | 0.452173913 | | -3.969754253 | | 0.023107638 | | -0.023107638 | | | 0.884037807 | 0.020428026 |
| 485 | 33.06 | | 0.360869565 | | -4.725897921 | | 0.023093884 | | -0.023093884 | | | 0.979629337 | 0.022623446 |
| 508 | 33.89 | | 0.252173913 | | -4.080025205 | | 0.023080138 | | -0.023080138 | | | 0.628834532 | 0.014513588 |
| 531 | 34.47 | | 0.158333333 | | -5.872584541 | | 0.0230664 | | -0.0230664 | | | -0.008500426 | -0.000196074 |
| 555 | 34.85 | | 0.017391304 | | -4.914933837 | | 0.023052074 | | -0.023052074 | | | -0.179647026 | -0.004141236 |
| 578 | 34.89 | | -0.095652174 | | -4.339233545 | | 0.023038352 | | -0.023038352 | | | -0.763800916 | -0.017596715 |
| 601 | 34.67 | | -0.195454545 | | -4.562672176 | | 0.023024639 | | -0.023024639 | | | -0.999914708 | -0.023022676 |
| 623 | 34.24 | | -0.295833333 | | -5.666035354 | | 0.02301153 | | -0.02301153 | | | -0.369876859 | -0.008511433 |
| 647 | 33.53 | | -0.431818182 | | -3.099173554 | | 0.022997238 | | -0.022997238 | | | -0.205279596 | -0.004720864 |
| 669 | 32.58 | | -0.5 | | -5.335968379 | | 0.022984144 | | -0.022984144 | | | 0.840514062 | 0.019318496 |
| 692 | 31.43 | | -0.622727273 | | -1.931369026 | | 0.022970463 | | -0.022970463 | | | 0.993280822 | 0.022816121 |
| 714 | 30.06 | | -0.665217391 | | -5.238995409 | | 0.022957385 | | -0.022957385 | | | 0.247236014 | 0.005675892 |
| 737 | 28.53 | | -0.785714286 | | -0.680272109 | | 0.02294372 | | -0.02294372 | | | -0.424787591 | -0.009746208 |
| 758 | 26.88 | | -0.8 | | -2.173913043 | | 0.022931251 | | -0.022931251 | | | -0.831308556 | -0.019062945 |
| 781 | 25.04 | | -0.85 | | -0.20661157 | | 0.022917601 | | -0.022917601 | | | -0.288733718 | -0.006617084 |
| 803 | 23.17 | | -0.854545455 | | -3.581267218 | | 0.022904553 | | -0.022904553 | | | 0.790776641 | 0.018112385 |
| 825 | 21.29 | | -0.933333333 | | 3.535353535 | | 0.022891512 | | -0.022891512 | | | 0.854472802 | 0.019560175 |
| 846 | 19.33 | | -0.859090909 | | -1.859504132 | | 0.022879071 | | -0.022879071 | | | -0.6418224 | -0.0146843 |
| 868 | 17.44 | | -0.9 | | 4.545454545 | | 0.022866045 | | -0.022866045 | | | -0.945699376 | -0.021624404 |
| 889 | 15.55 | | -0.804545455 | | 0.20661157 | | 0.022853617 | | -0.022853617 | | | 0.463258025 | 0.010587122 |
| 911 | 13.78 | | -0.8 | | 5.194805195 | | 0.022840606 | | -0.022840606 | | | 0.993296615 | 0.022687496 |
| 932 | 12.1 | | -0.690909091 | | 0.669027942 | | 0.022828192 | | -0.022828192 | | | -0.26332149 | -0.006011154 |
| 954 | 10.58 | | -0.676190476 | | 5.576169862 | | 0.022815195 | | -0.022815195 | | | -0.995068648 | -0.022702685 |
| 975 | 9.16 | | -0.559090909 | | 3.118850846 | | 0.022802795 | | -0.022802795 | | | 0.051236759 | 0.001168341 |
| 997 | 7.93 | | -0.49047619 | | 4.308390023 | | 0.022789812 | | -0.022789812 | | | 0.95093372 | 0.021671601 |
| 1018 | 6.9 | | -0.4 | | 4.988662132 | | 0.022777426 | | -0.022777426 | | | 0.163211753 | 0.003717544 |
| 1039 | 6.06 | | -0.295238095 | | 5.184498042 | | 0.022765047 | | -0.022765047 | | | -0.999911104 | -0.022763023 |
| 1060 | 5.44 | | -0.186363636 | | 5.007870917 | | 0.022752675 | | -0.022752675 | | | 0.136846476 | 0.003113623 |
| 1082 | 5.03 | | -0.076190476 | | 2.762317048 | | 0.02273972 | | -0.02273972 | | | 0.973998575 | 0.022148455 |
| 1103 | 4.87 | | -0.018181818 | | 6.670602125 | | 0.022727362 | | -0.022727362 | | | 0.077848308 | 0.001769287 |
| 1125 | 4.83 | | 0.128571429 | | 3.401360544 | | 0.022714422 | | -0.022714422 | | | -0.903151142 | -0.020514556 |
| 1146 | 5.1 | | 0.2 | | 6.493506494 | | 0.022702077 | | -0.022702077 | | | -0.288951602 | -0.006559802 |
| 1168 | 5.54 | | 0.342857143 | | 2.721088435 | | 0.022689151 | | -0.022689151 | | | 0.790637316 | 0.01793889 |
| 1189 | 6.26 | | 0.4 | | 6.493506494 | | 0.02267682 | | -0.02267682 | | | 0.486724268 | 0.011037359 |
| 1211 | 7.14 | | 0.542857143 | | -1.212658977 | | 0.022663909 | | -0.022663909 | | | -0.64164786 | -0.014542249 |
| 1232 | 8.28 | | 0.517391304 | | 6.283193807 | | 0.022651592 | | -0.022651592 | | | -0.662042171 | -0.014996309 |
| 1255 | 9.47 | | 0.661904762 | | 0.157744257 | | 0.022638109 | | -0.022638109 | | | -0.034979276 | -0.000791865 |
| 1276 | 10.86 | | 0.665217391 | | 3.48857192 | | 0.022625805 | | -0.022625805 | | | 0.993322897 | 0.022474731 |
| 1299 | 12.39 | | 0.745454545 | | 2.066115702 | | 0.022612338 | | -0.022612338 | | | 0.693703012 | 0.015686247 |
| 1321 | 14.03 | | 0.790909091 | | -1.167804527 | | 0.022599463 | | -0.022599463 | | | -0.424457915 | -0.009592521 |
| 1343 | 15.77 | | 0.765217391 | | 1.134215501 | | 0.022586596 | | -0.022586596 | | | -0.997369584 | -0.022527184 |
| 1366 | 17.53 | | 0.791304348 | | -0.56710775 | | 0.022573152 | | -0.022573152 | | | -0.72407678 | -0.016344695 |
| 1389 | 19.35 | | 0.77826087 | | 1.340436501 | | 0.022559716 | | -0.022559716 | | | -0.12084492 | -0.002726227 |
| 1412 | 21.14 | | 0.809090909 | | -3.575278476 | | 0.022546288 | | -0.022546288 | | | 0.537452176 | 0.012117551 |
| 1434 | 22.92 | | 0.730434783 | | -0.189035917 | | 0.022533451 | | -0.022533451 | | | 0.97974799 | 0.022077103 |
| 1457 | 24.6 | | 0.726086957 | | -3.024574669 | | 0.022520038 | | -0.022520038 | | | 0.629294506 | 0.014171736 |
| 1480 | 26.27 | | 0.656521739 | | -2.268431002 | | 0.022506634 | | -0.022506634 | | | -0.007908729 | -0.000177999 |
| 1503 | 27.78 | | 0.604347826 | | -3.213610586 | | 0.022493237 | | -0.022493237 | | | -0.641508204 | -0.014429596 |
| 1526 | 29.17 | | 0.530434783 | | -3.59168242 | | 0.022479849 | | -0.022479849 | | | -0.98279253 | -0.022093027 |
| 1549 | 30.39 | | 0.447826087 | | -3.709829868 | | 0.022466468 | | -0.022466468 | | | -0.876249155 | -0.019686224 |
| 1572 | 31.42 | | 0.3625 | | -3.361742424 | | 0.022453095 | | -0.022453095 | | | -0.370426548 | -0.008317223 |
| 1596 | 32.29 | | 0.281818182 | | -5.423553719 | | 0.02243915 | | -0.02243915 | | | -0.205858676 | -0.004619294 |
| 1618 | 32.91 | | 0.1625 | | -4.234601449 | | 0.022426374 | | -0.022426374 | | | 0.840193328 | 0.01884249 |
| 1642 | 33.3 | | 0.060869565 | | -4.347826087 | | 0.022412445 | | -0.022412445 | | | 0.920659561 | 0.020634232 |
| 1665 | 33.44 | | -0.039130435 | | -4.347826087 | | 0.022399104 | | -0.022399104 | | | 0.958948866 | 0.021479596 |
| 1688 | 33.35 | | -0.139130435 | | -4.820415879 | | 0.022385772 | | -0.022385772 | | | 0.560275279 | 0.012542195 |
| 1711 | 33.03 | | -0.25 | | -3.598484848 | | 0.022372447 | | -0.022372447 | | | -0.093698164 | -0.002096257 |
| 1733 | 32.48 | | -0.329166667 | | -5.034722222 | | 0.022359709 | | -0.022359709 | | | -0.963240935 | -0.021537787 |
| 1757 | 31.69 | | -0.45 | | -1.877470356 | | 0.022345822 | | -0.022345822 | | | -0.995018861 | -0.022234514 |
| 1779 | 30.7 | | -0.491304348 | | -4.330641004 | | 0.022333099 | | -0.022333099 | | | -0.449020037 | -0.010028009 |
| 1802 | 29.57 | | -0.590909091 | | -1.203736974 | | 0.022319806 | | -0.022319806 | | | 0.221044777 | 0.004933677 |
| 1824 | 28.27 | | -0.617391304 | | -5.247997119 | | 0.022307098 | | -0.022307098 | | | 0.989802887 | 0.02207963 |
| 1847 | 26.85 | | -0.738095238 | | 0.364783595 | | 0.02229382 | | -0.02229382 | | | 0.854803669 | 0.019056839 |
| 1868 | 25.3 | | -0.730434783 | | -4.887928706 | | 0.022281704 | | -0.022281704 | | | -0.641333604 | -0.014290005 |
| 1891 | 23.62 | | -0.842857143 | | 2.661934339 | | 0.022268441 | | -0.022268441 | | | -0.982750467 | -0.021884321 |
| 1912 | 21.85 | | -0.786956522 | | -3.0515798 | | 0.022256339 | | -0.022256339 | | | -0.035388674 | -0.000787622 |
| 1935 | 20.04 | | -0.857142857 | | 3.153988868 | | 0.022243091 | | -0.022243091 | | | 0.607697295 | 0.013517066 |
| 1956 | 18.24 | | -0.790909091 | | -0.846123573 | | 0.022231002 | | -0.022231002 | | | 0.69399801 | 0.015428271 |
| 1978 | 16.5 | | -0.80952381 | | 3.700267986 | | 0.022218345 | | -0.022218345 | | | -0.424086962 | -0.00942251 |
| 1999 | 14.8 | | -0.731818182 | | -0.068870523 | | 0.02220627 | | -0.02220627 | | | -0.831738385 | -0.018469807 |
| 2021 | 13.19 | | -0.733333333 | | 3.102453102 | | 0.022193626 | | -0.022193626 | | | 0.220911603 | 0.00490283 |
| 2042 | 11.65 | | -0.668181818 | | 2.233372688 | | 0.022181565 | | -0.022181565 | | | 0.931106956 | 0.020653409 |
| 2064 | 10.18 | | -0.619047619 | | 5.452484024 | | 0.022168935 | | -0.022168935 | | | -0.007544605 | -0.000167256 |
| 2085 | 8.88 | | -0.504545455 | | 4.102715466 | | 0.022156887 | | -0.022156887 | | | -0.987519405 | -0.021880356 |
| 2107 | 7.77 | | -0.414285714 | | 5.875077304 | | 0.022144272 | | -0.022144272 | | | -0.206170458 | -0.004565495 |
| 2128 | 6.9 | | -0.290909091 | | 2.08677686 | | 0.022132237 | | -0.022132237 | | | 0.998373176 | 0.022096232 |
| 2150 | 6.26 | | -0.245 | | 6.568181818 | | 0.022119636 | | -0.022119636 | | | 0.410373958 | 0.009077322 |
| 2170 | 5.77 | | -0.113636364 | | 4.94883904 | | 0.022108186 | | -0.022108186 | | | -0.970337365 | -0.021452399 |
| 2192 | 5.52 | | -0.004761905 | | 3.689960833 | | 0.022095599 | | -0.022095599 | | | -0.12134192 | -0.002681122 |
| 2213 | 5.51 | | 0.072727273 | | 4.486422668 | | 0.02208359 | | -0.02208359 | | | 0.999580412 | 0.022074324 |
| 2235 | 5.67 | | 0.171428571 | | 5.689548547 | | 0.022071017 | | -0.022071017 | | | 0.330512057 | 0.007294737 |
| 2256 | 6.03 | | 0.290909091 | | 3.719008264 | | 0.022059022 | | -0.022059022 | | | -0.982708353 | -0.021677585 |
| 2278 | 6.67 | | 0.372727273 | | 3.925619835 | | 0.022046462 | | -0.022046462 | | | -0.524434197 | -0.011561919 |
| 2300 | 7.49 | | 0.459090909 | | 4.45690673 | | 0.02203391 | | -0.02203391 | | | 0.60751654 | 0.013385965 |
| 2322 | 8.5 | | 0.557142857 | | -0.029577048 | | 0.022021365 | | -0.022021365 | | | 0.959064954 | 0.021119919 |
| 2343 | 9.67 | | 0.556521739 | | 5.82410658 | | 0.022009397 | | -0.022009397 | | | -0.423880846 | -0.009329362 |
| 2366 | 10.95 | | 0.69047619 | | -1.409839298 | | 0.021996296 | | -0.021996296 | | | -0.902818669 | -0.019858667 |
| 2387 | 12.4 | | 0.660869565 | | 0.945179584 | | 0.021984342 | | -0.021984342 | | | -0.289692293 | -0.006368694 |
| 2410 | 13.92 | | 0.682608696 | | 3.522942086 | | 0.021971256 | | -0.021971256 | | | 0.384483808 | 0.008447592 |
| 2433 | 15.49 | | 0.763636364 | | -1.509162774 | | 0.021958178 | | -0.021958178 | | | 0.883462708 | 0.019399231 |
| 2455 | 17.17 | | 0.730434783 | | 1.048290084 | | 0.021945676 | | -0.021945676 | | | 0.753526203 | 0.016536642 |
| 2478 | 18.85 | | 0.754545455 | | -0.700682716 | | 0.021932613 | | -0.021932613 | | | 0.164109814 | 0.003599357 |
| 2500 | 20.51 | | 0.739130435 | | -0.945179584 | | 0.021920126 | | -0.021920126 | | | -0.862467601 | -0.018905398 |
| 2523 | 22.21 | | 0.717391304 | | -2.268431002 | | 0.021907078 | | -0.021907078 | | | -0.987562381 | -0.021634606 |
| 2546 | 23.86 | | 0.665217391 | | -0.266368792 | | 0.021894039 | | -0.021894039 | | | -0.662655988 | -0.014508216 |
| 2569 | 25.39 | | 0.659090909 | | -5.148071625 | | 0.021881007 | | -0.021881007 | | | -0.035798065 | -0.000783298 |
| 2591 | 26.84 | | 0.545833333 | | -0.394570707 | | 0.021868549 | | -0.021868549 | | | 0.920428413 | 0.020128434 |
| 2615 | 28.15 | | 0.536363636 | | -4.683195592 | | 0.021854966 | | -0.021854966 | | | 0.973781577 | 0.021281963 |
| 2637 | 29.33 | | 0.433333333 | | -2.525252525 | | 0.021842523 | | -0.021842523 | | | 0.560765304 | 0.012248529 |
| 2661 | 30.37 | | 0.372727273 | | -4.6315427 | | 0.021828957 | | -0.021828957 | | | 0.41066449 | 0.008964377 |
| 2683 | 31.19 | | 0.270833333 | | -3.140782828 | | 0.021816528 | | -0.021816528 | | | -0.704556507 | -0.015370977 |
| 2707 | 31.84 | | 0.195454545 | | -5.85399449 | | 0.021802978 | | -0.021802978 | | | -0.815692399 | -0.017784523 |
| 2729 | 32.27 | | 0.066666667 | | -3.683574879 | | 0.021790564 | | -0.021790564 | | | -0.831990993 | -0.018129553 |
| 2753 | 32.43 | | -0.02173913 | | -3.59168242 | | 0.02177703 | | -0.02177703 | | | -0.724672992 | -0.015781226 |
| 2776 | 32.38 | | -0.104347826 | | -4.15879017 | | 0.021764068 | | -0.021764068 | | | -0.121703355 | -0.00264876 |
| 2799 | 32.14 | | -0.2 | | -4.347826087 | | 0.021751113 | | -0.021751113 | | | 0.536722679 | 0.011674316 |
| 2822 | 31.68 | | -0.3 | | -3.359683794 | | 0.021738166 | | -0.021738166 | | | 0.950581039 | 0.020663889 |
| 2844 | 31.02 | | -0.373913043 | | -4.493899295 | | 0.02172579 | | -0.02172579 | | | 0.629966379 | 0.013686517 |
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| 5813 | 22.27 | | 0.582608696 | | -0.56710775 | | 0.020118525 | | -0.020118525 | | | 0.754901033 | 0.015187495 |
| 5836 | 23.61 | | 0.569565217 | | -0.455404709 | | 0.02010655 | | -0.02010655 | | | 0.166174836 | 0.003341203 |
| 5859 | 24.92 | | 0.559090909 | | -3.443526171 | | 0.020094582 | | -0.020094582 | | | -0.498272017 | -0.010012568 |
| 5881 | 26.15 | | 0.483333333 | | -1.01010101 | | 0.020083141 | | -0.020083141 | | | -0.987889415 | -0.019839923 |
| 5905 | 27.31 | | 0.459090909 | | -4.958677686 | | 0.020070668 | | -0.020070668 | | | -0.946715557 | -0.019001213 |
| 5927 | 28.32 | | 0.35 | | -2.083333333 | | 0.02005924 | | -0.02005924 | | | -0.037890413 | -0.000760053 |
| 5951 | 29.16 | | 0.3 | | -1.976284585 | | 0.020046781 | | -0.020046781 | | | 0.133824912 | 0.002682759 |
| 5974 | 29.85 | | 0.254545455 | | -4.562672176 | | 0.020034849 | | -0.020034849 | | | 0.733040614 | 0.014686358 |
| 5996 | 30.41 | | 0.154166667 | | -2.635732323 | | 0.020023442 | | -0.020023442 | | | 0.897395344 | 0.017968944 |
| 6020 | 30.78 | | 0.090909091 | | -4.132231405 | | 0.020011006 | | -0.020011006 | | | 0.808561517 | 0.016180129 |
| 6042 | 30.98 | | 0 | | -3.409090909 | | 0.019999612 | | -0.019999612 | | | -0.260246432 | -0.005204828 |
| 6066 | 30.98 | | -0.081818182 | | -4.186130075 | | 0.01998719 | | -0.01998719 | | | -0.421777213 | -0.008430141 |
| 6088 | 30.8 | | -0.173913043 | | -3.703385461 | | 0.019975811 | | -0.019975811 | | | -0.997579668 | -0.019927463 |
| 6111 | 30.4 | | -0.259090909 | | -2.254761049 | | 0.01996392 | | -0.01996392 | | | -0.726114205 | -0.014496086 |
| 6133 | 29.83 | | -0.308695652 | | -3.969754253 | | 0.019952554 | | -0.019952554 | | | 0.382339858 | 0.007628657 |
| 6156 | 29.12 | | -0.4 | | -1.976284585 | | 0.019940678 | | -0.019940678 | | | 0.882372771 | 0.017595111 |
| 6178 | 28.24 | | -0.443478261 | | -3.050352294 | | 0.019929324 | | -0.019929324 | | | 0.755050272 | 0.015047642 |
| 6201 | 27.22 | | -0.513636364 | | -1.751706791 | | 0.019917462 | | -0.019917462 | | | 0.166399251 | 0.003314251 |
| 6223 | 26.09 | | -0.552173913 | | -2.672280461 | | 0.019906122 | | -0.019906122 | | | -0.861290412 | -0.017144952 |
| 6246 | 24.82 | | -0.613636364 | | -0.826446281 | | 0.019894273 | | -0.019894273 | | | -0.987924701 | -0.019654044 |
| 6268 | 23.47 | | -0.631818182 | | -2.272727273 | | 0.019882946 | | -0.019882946 | | | -0.208708478 | -0.004149739 |
| 6290 | 22.08 | | -0.681818182 | | 1.239669421 | | 0.019871626 | | -0.019871626 | | | 0.838610044 | 0.016664545 |
| 6312 | 20.58 | | -0.654545455 | | -1.416765053 | | 0.019860312 | | -0.019860312 | | | 0.808668634 | 0.016060411 |
| 6334 | 19.14 | | -0.685714286 | | 3.460514641 | | 0.019849004 | | -0.019849004 | | | -0.260070634 | -0.005162143 |
| 6355 | 17.7 | | -0.613043478 | | 0.764736209 | | 0.019838216 | | -0.019838216 | | | -0.915637055 | -0.018164606 |
| 6378 | 16.29 | | -0.595454545 | | 0.442739079 | | 0.019826408 | | -0.019826408 | | | -0.451580411 | -0.008953218 |
| 6400 | 14.98 | | -0.585714286 | | 2.566481138 | | 0.01981512 | | -0.01981512 | | | 0.671658308 | 0.01330899 |
| 6421 | 13.75 | | -0.531818182 | | 2.312081858 | | 0.019804351 | | -0.019804351 | | | 0.631696928 | 0.012510348 |
| 6443 | 12.58 | | -0.480952381 | | 3.638425067 | | 0.019793075 | | -0.019793075 | | | -0.497956272 | -0.009856086 |
| 6464 | 11.57 | | -0.404545455 | | 2.154663518 | | 0.019782318 | | -0.019782318 | | | -0.782670445 | -0.015483036 |
| 6486 | 10.68 | | -0.357142857 | | 5.318491033 | | 0.019771055 | | -0.019771055 | | | 0.30128129 | 0.005956649 |
| 6507 | 9.93 | | -0.245454545 | | 1.63321527 | | 0.019760309 | | -0.019760309 | | | 0.897535881 | 0.017735587 |
| 6529 | 9.39 | | -0.20952381 | | 4.566068852 | | 0.019749059 | | -0.019749059 | | | -0.090706856 | -0.001791375 |
| 6550 | 8.95 | | -0.113636364 | | 3.650137741 | | 0.019738326 | | -0.019738326 | | | -0.970993979 | -0.019165795 |
| 6572 | 8.7 | | -0.033333333 | | 3.03657695 | | 0.019727087 | | -0.019727087 | | | -0.124052289 | -0.00244719 |
| 6593 | 8.63 | | 0.030434783 | | 5.509046719 | | 0.019716366 | | -0.019716366 | | | 0.99965579 | 0.019709579 |
| 6616 | 8.7 | | 0.157142857 | | 1.82436611 | | 0.01970463 | | -0.01970463 | | | 0.755229307 | 0.014881514 |
| 6637 | 9.03 | | 0.195454545 | | 3.88626525 | | 0.019693921 | | -0.019693921 | | | -0.761357025 | -0.014994105 |
| 6659 | 9.46 | | 0.280952381 | | 1.735186828 | | 0.019682708 | | -0.019682708 | | | -0.877779988 | -0.017277088 |
| 6680 | 10.05 | | 0.317391304 | | 4.212800432 | | 0.019672011 | | -0.019672011 | | | 0.60534502 | 0.011908354 |
| 6703 | 10.78 | | 0.414285714 | | 0.769003253 | | 0.019660302 | | -0.019660302 | | | 0.973198569 | 0.019133378 |
| 6724 | 11.65 | | 0.430434783 | | 4.059771357 | | 0.019649617 | | -0.019649617 | | | 0.081341976 | 0.001598339 |
| 6747 | 12.64 | | 0.523809524 | | -0.305629498 | | 0.019637921 | | -0.019637921 | | | -0.570506117 | -0.011203554 |
| 6768 | 13.74 | | 0.517391304 | | 1.615397835 | | 0.019627248 | | -0.019627248 | | | -0.726395809 | -0.014257151 |
| 6791 | 14.93 | | 0.554545455 | | 0.287459576 | | 0.019615565 | | -0.019615565 | | | -0.124187784 | -0.002436014 |
| 6813 | 16.15 | | 0.560869565 | | -0.077332875 | | 0.019604397 | | -0.019604397 | | | 0.882179935 | 0.017294606 |
| 6836 | 17.44 | | 0.559090909 | | 0.278476464 | | 0.019592728 | | -0.019592728 | | | 0.980416869 | 0.019209041 |
| 6858 | 18.67 | | 0.565217391 | | 0.721773501 | | 0.019581573 | | -0.019581573 | | | 0.166803177 | 0.003266269 |
| 6881 | 19.97 | | 0.581818182 | | -1.742723679 | | 0.019569917 | | -0.019569917 | | | -0.49771942 | -0.009740328 |
| 6903 | 21.25 | | 0.543478261 | | 0.481182334 | | 0.019558775 | | -0.019558775 | | | -0.987988087 | -0.019323837 |
| 6926 | 22.5 | | 0.554545455 | | -23.65482427 | | 0.019547133 | | -0.019547133 | | | -0.664698829 | -0.012992956 |
| 6948 | 23.72 | | 0.034139321 | | 0.004913546 | | 0.019536004 | | -0.019536004 | | | 0.459906672 | 0.008984739 |
| t1 | | t2 | | t3 | | T1 secs | | T2 secs | | TD |
| 0.606 | | 1.147 | | 1.695 | | 0.541 | | 0.548 | | 1.089 |
| ωD=2π/TD | | 5.7667585 | |  | |  | |  | |  |
| u1 | | u6 | | Δ | | ζ | |  | |  |
| 29.32 | | 25.32 | | 0.0293351 | | 0.004489 | |  | |  |
| ωn=ωD/SQRT(1-zeta\*zeta) | | 5.7668166 | |  | |  | |  | |  |
| K=mω2 | | 11.639661 | |  | |  | |  | |  |
| u(0) m | |  | |  | |  | |  | |  |
| 0.0185 | |  | |  | |  | |  | |  |
| v(0) m/s | |  | |  | |  | |  | |  |
| 0.08201581 | |  | |  | |  | |  | |  |
| A2 | |  | |  | |  | |  | |  |
| 0.00034225 | |  | |  | |  | |  | |  |
| B | |  | |  | |  | |  | |  |
| 0.014305215 | |  | |  | |  | |  | |  |
| B2 | |  | |  | |  | |  | |  |
| 0.000204639 | |  | |  | |  | |  | |  |
| uo=SQRT(A2+B2) | |  | |  | |  | |  | |  |
| 0.023385662 | |  | |  | |  | |  | |  |
| φo=ATAN(-A/B) | |  | |  | |  | |  | |  |
| -0.912577448 | |  | |  | |  | |  | |  |

# 

**Conclusions**:

Hence, we successfully calculated damped time period, angular frequency, frequency, logarithmic decrement, damping ratio, natural frequency, initial displacement, initial velocity, initial phase, amplitude, exponentially decaying envelop, u(t), v(t) and a(t)

**Questions**:

Q1. A di-atomic molecule can be modelled as two atoms with masses m1 and m2 attached to the two ends of a spring with spring constant k. If the molecule is set into vibration the its angular

frequency of vibration is given by 𝜔 = √𝑘, where 𝜇 = 𝑚1𝑚2 is the reduced mass of the system.

𝜇 𝑚1+𝑚2

If the frequency of electromagnetic radiation from HCl gas is observed to be 𝑓 = 8.969 × 1013𝐻𝑧, what is the spring constant of the bond between H and Cl atoms. Please find the masses of the H and Cl atoms from the internet.

